Three different types of Willmore flows and their analytic properties

Ruben Jakob

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Abstract

In this talk we consider the classical Willmore flow

$$\partial_t f_t = -\frac{1}{2}\left(\triangle^{\perp}_{f_t} H_{f_t} + Q(A^0_{f_t}).H_{f_t}\right) \equiv -\nabla_{L^2}\mathcal{W}(f_t), \tag{1}$$

i.e. the L^2-gradient flow of the Willmore functional

$$\mathcal{W}(f) := \frac{1}{4}\int_{\Sigma} \mid H_f \mid^2 \, d\mu_f \tag{2}$$

for smooth immersions $f : \Sigma \longrightarrow \mathbb{R}^n$ from an abstract compact surface into Euclidean space $\mathbb{R}^n$, and two natural modifications of flow (1), namely the inverse Willmore flow

$$\partial_t f_t = -\frac{1}{2}\,|f_t|^8 \left(\triangle^{\perp}_{f_t} H_{f_t} + Q(A^0_{f_t}).H_{f_t}\right) =: -|f_t|^8\,\nabla_{L^2}\mathcal{W}(f_t),$$

and the conformally invariant Willmore flow:

$$\partial_t f_t = -\frac{1}{2}\frac{1}{|A^0_{f_t}|^4}\left(\triangle^{\perp}_{f_t} H_{f_t} + Q(A^0_{f_t}).H_{f_t}\right) =: -\frac{1}{|A^0_{f_t}|^4}\,\nabla_{L^2}\mathcal{W}(f_t).$$

Here "H_{f_t}" denotes the traces of the second fundamental forms A_{f_t} of the flowing immersions f_t, and "$A^0_{f_t}$" denotes the trace-free parts of A_{f_t}. We will recall prominent results about the classical Willmore flow (1) due to Kuwert and Schätzle from 2001–2004 and compare them to new results about the inverse and conformally invariant Willmore flow due to M. Mayer and the speaker.

*Institute: Mathematics Department, Technion: Israel Institute of Technology, 3200003 Haifa, Israel. Phone: 00972 549298486. E-mail: rubenj at technion.ac.il