Forschungsinstitut für Mathematik

ETH Zürich

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Special Lectures

April 8, 2009, 15:45–17:00, HG G43 (HWZ)

Scl, sails and surgery

Abstract: Given a group G and an element $g \in [G, G]$, the *commutator length* of g, denoted $\mathrm{cl}(g)$, is the smallest number of commutators in G whose product is g, and the *stable commutator length* of g is the limit $\mathrm{scl}(g) := \lim_{n \to \infty} \mathrm{cl}(g^n)/n$. Commutator length in a group extends in a natural way to a pseudo-norm on the real vector space of 1-boundaries (in group homology). We show that the problem of computing stable commutator length in free products of abelian groups reduces to a (finite dimensional) integer programming problem; moreover, families of elements in such groups give rise to families of integer programming problems that are related in explicit ways. Thus stable commutator length in free groups is naturally related to the theory of multi-dimensional continued fractions and Klein polyhedra. This gives rise to several interesting phenomena in the scl spectrum of a free group, some of which are understood, and some of which are conjectural.

April 22, 2009, 15:45–17:00, HG G43 (HWZ)

Faces of the scl norm ball

Abstract: If G is a hyperbolic group, stable commutator length defines a norm on $B_1^H(G)$, the space of homogenized group 1-boundaries. When G is a free group, the unit ball in this norm is a rational polyhedron. We show that every rational chain projectively intersects the interior of an infinite dimensional face of this norm ball. One can think of this as a kind of rigidity result, related to theorems of Goldman, Matsumoto, and Burger-Iozzi-Wienhard.