Hopf-Sullivan Dichotomy in non positive curvature

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Abstract : Certainly, one remarkable characteristic of the geodesic flow on a compact manifold with negative curvature is his hyperbolic behavior expressed by Anosov property. This property is a starting point from which deep results can be achieved at the interplay between dynamics and geometry. For instance, it implies that the geodesic flow is ergodic with respect to Liouville measure or with respect to maximal entropy's measure ; furthermore the latter is unique. Ergodicity can't be true in full generality, namely when the manifold M is not assumed to be of finite volume, and to study such a problem, the weakest and natural assumption to consider for the manifold M is a topological one, namely that $\Gamma := \Pi_1(M)$ is non-elementary. In that case and when the curvature is still bounded from above by a negative constant, D. Sullivan, using the famous E. Hopf's argument observed first in constant negative curvature for Liouville measure that a dichotomy occurs : either the geodesic flow is totally dissipative (there exists *only* wandering sets) or the geodesic flow is totally conservative (there exists *no* wandering sets) and ergodic with respect to a suitable class of measures (a conformal density). Ergodicity is equivalent in this context to the recurrence of Brownian motion, hence the dichotomy is called Hopf-Tsuji-Sullivan by specialists, HTS for short, the proof of which can be extended in various hyperbolic frameworks such as $CAT(-1)$ spaces.

We prove with G. Link that HTS dichotomy is still valid for non-elementary *rank one manifolds*, that is manifolds whose first fundamental group Γ contains an axial element translating a unique geodesic σ in the universal covering, the geodesic σ being of rank one in the sense that it does not admit a perpendicular parallel Jacobi field. We will see along the lines of the talk that all the results but Hopf's argument are valid with the weaker assumption that there exists a rank one element $\gamma \in \Gamma$ in the sense that γ acts by translation on a geodesic in the universal covering that does not bound a flat half plane. This condition can be considered as a dividing line between rank one manifolds and higher rank locally symmetric spaces.