ULTRARIGID TANGENTS OF SUB-RIEMANNIAN NILPOTENT GROUPS

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Abstract. In [MM95], Margulis and Mostow show that if two equiregular sub-Riemannian manifolds are quasiconformally equivalent, then almost everywhere they have isomorphic Gromov tangent cones. In other words, the tangent cone is a quasiconformal (and therefore bi-Lipschitz) invariant. Their work extends a result [Pan89] of Pansu, for which two Carnot groups are quasiconformally equivalent only if they are isomorphic.

Part one. In the first talk we discuss the state of the art and present a joint result [LOW] with E. Le Donne and B. Warhurst showing that the converse of the theorem of Margulis and Mostow fails in a strong sense. Namely, we show that there exist two nilpotent Lie groups equipped with left invariant sub-Riemannian metrics whose tangent cones are isomorphic at every point but which are not quasiconformally equivalent.

This result relies on studying those Carnot groups whose quasiconformal maps are only translations and dilations. We shall refer to groups with this property as ultrarigid groups. One of the tools in our method is to provide an algebraic characterization of ultrarigidity.

Part two. In the second seminar we shall give more details on the tools and the methods that come into play in what we discussed in the first part.

We recall the definition of tangent cone at a point of a metric space. Also, we define the nilpotentisation of a sub-Riemannian connected and simply connected Lie group G, following a construction given in [RoSt] and [NRS]. Then we state a theorem of Mitchell, claiming that the tangent cone at every point to G coincides with its nilpotentisation.

Finally, if we have time we shall present some of the proofs of the results presented in part one.

References

[LOW] Enrico Le Donne, Alessandro Ottazzi and Ben Warhurst, *Ultrarigid tangents of sub-Riemannian nilpotent groups*, arXiv:1104.3071v1 (2011), submitted.

[MM95] Gregori A. Margulis and George D. Mostow, *The differential of a quasi-conformal mapping of a Carnot-Carathéodory space*, Geom. Funct. Anal. **5** (1995), no. 2, 402–433.

[NRS] Nagel, A. and Ricci, F. and Stein, E. M., *Harmonic analysis and fundamental solutions on nilpotent Lie groups*, Analysis and partial differential equations, Lecture Notes in Pure and Appl. Math., 122, 249–275, Dekker, New York, 1990,

[Pan89] Pierre Pansu, *Métriques de Carnot-Carathéodory et quasiisométries des espaces symétriques de rang un*, Ann. of Math. (2) **129** (1989), no. 1, 1–60.

[RoSt] Rothschild, Linda Preiss and Stein, E. M., *Hypoelliptic differential operators and nilpotent groups*, Acta Math., 137, 1976, 3-4, 247–320,

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