Moments, model reduction and nonlinearity in solving linear algebraic problems

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Krylov subspace methods play an important role in many areas of scientific computing, including numerical solution of linear algebraic systems arising from discretisation of partial differential or integral equations. By their nature they represent *model reductions based on matching moments*. Such view naturally complements, in our opinion, the standard description using the projection processes framework, and it shows their highly nonlinear character.

We present three examples that link algebraic views of problems with views from related areas of mathematics:

- Matching moments reduced order modeling in approximation of large-scale linear dynamical systems is linked with the classical work on moments and continued fractions by Chebyshev and Stieltjes, and with development of the conjugate gradient method by Hestenes and Stiefel.
- We show that Gauss-Christoffel quadrature for a small number of quadrature nodes can be highly sensitive to small changes in the distribution function, and we relate the sensitivity of Gauss-Christoffel quadrature to the convergence properties of the CG and Lanczos methods in exact and in finite precision arithmetic.
- Based on the method of moments, we show how the information from the Golub-Kahan iterative bidiagonalization can be used for estimating the noise level in discrete ill-posed problems.

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